Graph matroid families

Dániel Garamvölgyi (Eötvös Loránd University, Budapest)

Rigid structures in algebraic combinatorics and algebraic statistics

ICMS, May 21, 2026

Where I come from

Whitney's theorem (weak form)

Every 3-connected graph is uniquely determined by its graphic matroid.

Where I come from

Whitney's theorem (weak form)

Every 3-connected graph is uniquely determined by its graphic matroid.

Similar results for:

- bicircular matroids (Wagner 1985)

Where I come from

Whitney's theorem (weak form)

Every 3-connected graph is uniquely determined by its graphic matroid.

Similar results for:

- bicircular matroids (Wagner 1985)
- the 2d generic rigidity matroids (Jordán-Kaszanitzky 2013)

Where I come from

Whitney's theorem (weak form)

Every 3-connected graph is uniquely determined by its graphic matroid.

Similar results for:

- bicircular matroids (Wagner 1985)
- the 2d generic rigidity matroids (Jordán-Kaszanitzky 2013)
- (k, ℓ)-count matroids (G.-Jordán-Király 2024)
- (unions of) the $\mathcal{C}_2^1$-cofactor matroid (G.-Jordán-Király 2024)

Where I come from

Whitney's theorem (weak form)

Every 3-connected graph is uniquely determined by its graphic matroid.

Similar results for:

- bicircular matroids (Wagner 1985)
- the 2d generic rigidity matroids (Jordán-Kaszanitzky 2013)
- (k,ℓ)-count matroids (G.-Jordán-Király 2024)
- (unions of) the $\mathcal{C}_2^1$-cofactor matroid (G.-Jordán-Király 2024)
- the d-dimensional generic rigidity matroid (G. 2025)

Graph matroid families

Definition

A **graph matroid family** $\mathcal{M}$ is an assignment

$$G \mapsto \mathcal{M}(G) = (E(G), \mathcal{I}_G)$$

of a matroid to each finite graph satisfying:

- if $G \cong G'$, then $\mathcal{M}(G) \cong \mathcal{M}(G')$, “well-defined”
- if $H \subseteq G$, then $\mathcal{M}(H) = \mathcal{M}(G)|_{E(H)}$. “compatible”

Definition (G. 2025)

A **graph matroid family** $\mathcal{M}$ is an assignment

$$G \mapsto \mathcal{M}(G) = (E(G), \mathcal{I}_G)$$

that is well-defined and compatible.

Or equivalently:

Definition (G. 2025)

A **graph matroid family** $\mathcal{M}$ is an assignment

$$G \mapsto \mathcal{M}(G) = (E(G), \mathcal{I}_G)$$

that is well-defined and compatible.

Or equivalently:

Definition (Király-Rosen-Theran 2013)

A **graph matroid limit** is a direct limit of an injective sequence of graph matroids.

Definition (G. 2025)

A **graph matroid family** $\mathcal{M}$ is an assignment

$$G \mapsto \mathcal{M}(G) = (E(G), \mathcal{I}_G)$$

that is well-defined and compatible.

Or equivalently:

Definition (Kalai 1990)

A **2-symmetric matroid** is a (finitary) matroid on the edge set of the countable complete graph $K_\mathbb{N}$ that is invariant under automorphisms of $K_\mathbb{N}$.

Definition (G. 2025)

A **graph matroid family** $\mathcal{M}$ is an assignment

$$G \mapsto \mathcal{M}(G) = (E(G), \mathcal{I}_G)$$

that is well-defined and compatible.

Or equivalently*:

Definition (Simões-Pereira 1972)

A **matroidal family** is a set $\mathcal{C}$ of (connected) graphs such that for every graph G,

$$\{E(C) : C \subseteq G, C \text{ is isomorphic to some } C' \in \mathcal{C}\}$$

is the set of circuits of a matroid on $E(G)$.

Definition

A **graph matroid family** $\mathcal{M}$ is an assignment

$$G \mapsto \mathcal{M}(G) = (E(G), \mathcal{I}_G)$$

that is well-defined and compatible.

For a graph G,

- $r_{\mathcal{M}}(G) = \operatorname{rank}(\mathcal{M}(G))$,
- G is **$\mathcal{M}$-independent** $\Leftrightarrow$ $\mathcal{M}(G)$ is free,
- G is an **$\mathcal{M}$-circuit** $\Leftrightarrow$ $\mathcal{M}(G)$ is a circuit,
- G is **$\mathcal{M}$-rigid** $\Leftrightarrow$ $r_{\mathcal{M}}(G) = r_{\mathcal{M}}(K_{V(G)})$.
  (Motto: "G has maximal $\mathcal{M}$-rank on its vertex set")

Basic example: graphic matroid

The family $\mathcal{G}$ of **graphic matroids** is defined by

$$\mathcal{G}(G) = \text{graphic matroid of } G.$$

We have

- $r_{\mathcal{G}}(G) = |V(G)| - c(G)$,
- G is $\mathcal{G}$-independent $\Leftrightarrow$ G is a forest,
- G is a $\mathcal{G}$-circuit $\Leftrightarrow$ G is a cycle graph,
- G is $\mathcal{G}$-rigid $\Leftrightarrow$ G is connected.

Basic example: uniform matroids

The family $\mathcal{U}_k$ of **rank-k uniform matroids** is defined by

$$\mathcal{U}_k(G) = \text{rank-}k \text{ uniform matroid on } E(G).$$

We have

- $r_{\mathcal{U}_k}(G) = \min(k, |E(G)|)$,
- G is $\mathcal{U}_k$-independent $\Leftrightarrow |E(G)| \leq k$,
- G is a $\mathcal{U}_k$-circuit $\Leftrightarrow |E(G)| = k + 1$,
- G is $\mathcal{U}_k$-rigid $\Leftrightarrow |E(G)| \geq k$.

Non-examples

Definition

A **graph matroid family** $\mathcal{M}$ is an assignment

$$G \mapsto \mathcal{M}(G) = (E(G), \mathcal{I}_G)$$

of a matroid to each finite graph satisfying:

- if $G \cong G'$, then $\mathcal{M}(G) \cong \mathcal{M}(G')$, "well-defined"
- if $H \subseteq G$, then $\mathcal{M}(H) = \mathcal{M}(G)|_{E(H)}$. "compatible"

What about...

$$\mathcal{M}(G) = \text{cographic matroid of } G?$$

Non-examples

Definition

A **graph matroid family** $\mathcal{M}$ is an assignment

$$G \mapsto \mathcal{M}(G) = (E(G), \mathcal{I}_G)$$

of a matroid to each finite graph satisfying:

- if $G \cong G'$, then $\mathcal{M}(G) \cong \mathcal{M}(G')$, "well-defined"
- if $H \subseteq G$, then $\mathcal{M}(H) = \mathcal{M}(G)|_{E(H)}$. "compatible"

What about...

$$\mathcal{M}(G) = \text{cographic matroid of } G?$$

→ not compatible!

Non-examples

Definition

A **graph matroid family** $\mathcal{M}$ is an assignment

$$G \mapsto \mathcal{M}(G) = (E(G), \mathcal{I}_G)$$

of a matroid to each finite graph satisfying:

- if $G \cong G'$, then $\mathcal{M}(G) \cong \mathcal{M}(G')$, "well-defined"
- if $H \subseteq G$, then $\mathcal{M}(H) = \mathcal{M}(G)|_{E(H)}$. "compatible"

What about...

$$\mathcal{M}(G) = \text{rank-}|V(G)| \text{ uniform matroid on } E(G)?$$

Non-examples

Definition

A **graph matroid family** $\mathcal{M}$ is an assignment

$$G \mapsto \mathcal{M}(G) = (E(G), \mathcal{I}_G)$$

of a matroid to each finite graph satisfying:

- if $G \cong G'$, then $\mathcal{M}(G) \cong \mathcal{M}(G')$, "well-defined"
- if $H \subseteq G$, then $\mathcal{M}(H) = \mathcal{M}(G)|_{E(H)}$. "compatible"

What about...

$$\mathcal{M}(G) = \text{rank-}|V(G)| \text{ uniform matroid on } E(G)?$$

→ also not compatible!

Non-examples

Definition

A **graph matroid family** $\mathcal{M}$ is an assignment

$$G \mapsto \mathcal{M}(G) = (E(G), \mathcal{I}_G)$$

of a matroid to each finite graph satisfying:

- if $G \cong G'$, then $\mathcal{M}(G) \cong \mathcal{M}(G')$, "well-defined"
- if $H \subseteq G$, then $\mathcal{M}(H) = \mathcal{M}(G)|_{E(H)}$. "compatible"

What about... matroids defined using additional structure?
(matroids of group-labeled graphs, oriented graphs, etc.)

Non-examples

Definition

A **graph matroid family** $\mathcal{M}$ is an assignment

$$G \mapsto \mathcal{M}(G) = (E(G), \mathcal{I}_G)$$

of a matroid to each finite graph satisfying:

- if $G \cong G'$, then $\mathcal{M}(G) \cong \mathcal{M}(G')$, "well-defined"
- if $H \subseteq G$, then $\mathcal{M}(H) = \mathcal{M}(G)|_{E(H)}$. "compatible"

What about... matroids defined using additional structure?
(matroids of group-labeled graphs, oriented graphs, etc.)

→ not well-defined!

Dimension

The family $\mathcal{M}$ is **trivial** if $\mathcal{M}(G)$ is free for every graph G.

Dimension

The family $\mathcal{M}$ is **trivial** if $\mathcal{M}(G)$ is free for every graph G.

Dimension lemma (Kalai 1990, Király-Rosen-Theran 2013, G. 2026 ...)

For every nontrivial graph matroid family $\mathcal{M}$ there exist integers d, t, c such that

$$r_{\mathcal{M}}(K_n) = dn - c$$

for every $n \geq t$.

Dimension

The family $\mathcal{M}$ is **trivial** if $\mathcal{M}(G)$ is free for every graph G.

Dimension lemma (Kalai 1990, Király-Rosen-Theran 2013, G. 2026 ...)

For every nontrivial graph matroid family $\mathcal{M}$ there exist integers d, t, c such that

$$r_{\mathcal{M}}(K_n) = dn - c$$

for every $n \geq t$.

Definition

The (unique) number $d \geq 0$ is the **dimension** of $\mathcal{M}$.

We have $c \leq \binom{d+1}{2}$. (c may be negative!)

The following are equivalent:

- $\mathcal{M}$ is d-dimensional,
- $r_{\mathcal{M}}(K_n) = dn - c$ for large enough n, ← definition
- $d =$ largest k for which "degree-k vertex addition" preserves $\mathcal{M}$-independence,
- $d =$ smallest k for which there exists an $\mathcal{M}$-circuit with minimum degree $k + 1$.

The following are equivalent:

- $\mathcal{M}$ is d-dimensional,
- $r_{\mathcal{M}}(K_n) = dn - c$ for large enough n, ← definition
- $d =$ largest k for which "degree-k vertex addition" preserves $\mathcal{M}$-independence,
- $d =$ smallest k for which there exists an $\mathcal{M}$-circuit with minimum degree $k + 1$.

The $d = 0$ and the $d \geq 1$ cases behave very differently!

Definition

$\mathcal{M}$ is **bounded** if $d = 0$, and **unbounded** otherwise.

Examples

One-dimensional families

We saw:

- Family $\mathcal{G}$ of graphic matroids:

  $\mathcal{G}$-independent graphs $\iff$ forests. .

We also have:

- Family $\mathcal{B}$ of bicircular matroids:

  $\mathcal{B}$-independent graphs $\iff$ pseudoforests, .
- Family $\mathcal{E}$ of even cycle matroids:

  $\mathcal{E}$-independent graphs $\iff$ pseudoforests without cycles of even length.

Theorem (Simões-Pereira 1975)

If $\mathcal{M}$ is a one-dimensional graph matroid family for which every $\mathcal{M}$-circuit is connected, then $\mathcal{M} \in \{\mathcal{G}, \mathcal{B}, \mathcal{E}\}$.

Simões-Pereira: can we classify all graph matroid families?

Count matroids

Fix $k \geq 1$ and $\ell \leq 2k-1$. A graph G is (k,ℓ)-**sparse** if

$$\#\text{ of edges induced by } X \leq k|X| - \ell$$

for every $X \subseteq V(G)$ with $|X| \geq 2$.

Definition

The family $\mathcal{M}_{k,\ell}$ of (k,ℓ)-**count matroids** is defined by

$$G \text{ is } \mathcal{M}_{k,\ell}\text{-independent} \Leftrightarrow G \text{ is } (k,\ell)\text{-sparse.}$$

We have $r_{\mathcal{M}_{k,\ell}}(K_n) = kn - \ell$ for large n, so $\dim(\mathcal{M}_{k,\ell}) = k$.

Schmidt's construction

Fix $k \geq 2$ and let $\mathcal{C}$ be a set of 3-connected $2k$-regular graphs.

A graph is **$\mathcal{C}$-free** if it contains no copy of a graph from $\mathcal{C}$.

Proposition/Definition (Schmidt 1979)

The condition

$$G \text{ is } \mathcal{M}_{\mathcal{C}}\text{-independent} \Leftrightarrow G \text{ is } (k,0)\text{-sparse and } \mathcal{C}\text{-free}$$

defines a graph matroid family $\mathcal{M}_{\mathcal{C}}$.

Schmidt's construction

Fix $k \geq 2$ and let $\mathcal{C}$ be a set of 3-connected $2k$-regular graphs.

A graph is **$\mathcal{C}$-free** if it contains no copy of a graph from $\mathcal{C}$.

Proposition/Definition (Schmidt 1979)

The condition

$$G \text{ is } \mathcal{M}_{\mathcal{C}}\text{-independent} \Leftrightarrow G \text{ is } (k,0)\text{-sparse and } \mathcal{C}\text{-free}$$

defines a graph matroid family $\mathcal{M}_{\mathcal{C}}$.

$\Rightarrow$ there are uncountably many graph matroid families!.

Generic rigidity

Definition

The family $\mathcal{R}_d$ of d-**dimensional generic rigidity matroids** is defined by

$$\mathcal{R}_d(G) = \text{row matroid of } R_d(G,p)$$

for generic $p : V(G) \to \mathbb{R}^d$.

$$R_d(G,p) \;=\; \begin{array}{c c} & \begin{array}{ccccc} & u & & v & \end{array} \\ uv & \left(\begin{array}{ccccc} \cdots & p(u)-p(v) & \cdots & p(v)-p(u) & \cdots \end{array}\right) \end{array}$$

Generic rigidity

Definition

The family $\mathcal{R}_d$ of d-**dimensional generic rigidity matroids** is defined by

$$\mathcal{R}_d(G) = \text{row matroid of } R_d(G, p)$$

for generic $p : V(G) \to \mathbb{R}^d$.

Some properties:

- $r_{\mathcal{R}_d}(K_n) = dn - \binom{d+1}{2}$ for $n \geq d + 1$,
- K_{d+2} is an $\mathcal{R}_d$-circuit,
- every $\mathcal{R}_d$-rigid graph is d-connected.

Hyperconnectivity

Definition (Kalai 1985)

The family $\mathcal{H}_d$ of **d-dimensional hyperconnectivity matroids** is defined by

$$\mathcal{H}_d(G) = \text{row matroid of } H_d(\vec{G}, p)$$

for some orientation $\vec{G}$ and generic $p : V(G) \to \mathbb{R}^d$.

$$H_d(\vec{G}, p) \quad = \quad \begin{array}{c} \\ uv \\ \\ \end{array} \begin{array}{c} \begin{array}{ccccc} & u & & v & \end{array} \\ \left(\begin{array}{ccccc} & & & & \\ \cdots & p(v) & \cdots & -p(u) & \cdots \\ & & & & \end{array} \right) \end{array}$$

Hyperconnectivity

Definition (Kalai 1985)

The family $\mathcal{H}_d$ of **d-dimensional hyperconnectivity matroids** is defined by

$$\mathcal{H}_d(G) = \text{row matroid of } H_d(\vec{G}, p)$$

for some orientation $\vec{G}$ and generic $p : V(G) \to \mathbb{R}^d$.

Some properties:

- $r_{\mathcal{H}_d}(K_n) = dn - \binom{d+1}{2}$ for $n \geq d+1$,
- K_{d+2} and $K_{d+1,d+1}$ are $\mathcal{H}_d$-circuits,
- every $\mathcal{H}_d$-rigid graph is d-connected.

Rigidity families

Lemma

TFAE for a graph matroid family $\mathcal{M}$:

- $r_{\mathcal{M}}(K_n) = dn - \binom{d+1}{2}$ for $n \geq d+1$,
- $\mathcal{M}$ is d-dimensional and K_{d+2} is an $\mathcal{M}_d$-circuit,
- $\mathcal{M}(K_n)$ is a d-dimensional abstract rigidity matroid for all $n \geq d+1$.

Definition

If these hold, we say that $\mathcal{M}$ is a **d-rigidity family**.

Motto:

“a matroidal version of d-connectivity”

Three well-known constructions: $\mathcal{R}_d$, $\mathcal{H}_d$, and $\mathcal{C}_{d-1}^{d-2}$.

Proposition

The only 1-rigidity family is the family of graphic matroids.

Three well-known constructions: $\mathcal{R}_d$, $\mathcal{H}_d$, and $\mathcal{C}_{d-1}^{d-2}$.

Proposition

The only 1-rigidity family is the family of graphic matroids.

Conjecture

The only 2-rigidity families are $\mathcal{R}_2$ and $\mathcal{H}_2$.

Three well-known constructions: $\mathcal{R}_d$, $\mathcal{H}_d$, and $\mathcal{C}_{d-1}^{d-2}$.

Proposition

The only 1-rigidity family is the family of graphic matroids.

Conjecture / Wishful thinking?

The only 2-rigidity families are $\mathcal{R}_2$ and $\mathcal{H}_2$.

Three well-known constructions: $\mathcal{R}_d$, $\mathcal{H}_d$, and $\mathcal{C}_{d-1}^{d-2}$.

Proposition

The only 1-rigidity family is the family of graphic matroids.

Conjecture / Wishful thinking?

The only 2-rigidity families are $\mathcal{R}_2$ and $\mathcal{H}_2$.

Theorem (Tyomkyn 2026+)

The only 2-rigidity family $\mathcal{M}$ for which $K_{3,3}$ is $\mathcal{M}$-rigid is $\mathcal{M} = \mathcal{R}_2$.

Three well-known constructions: $\mathcal{R}_d$, $\mathcal{H}_d$, and $\mathcal{C}_{d-1}^{d-2}$.

Proposition

The only 1-rigidity family is the family of graphic matroids.

Conjecture / Wishful thinking?

The only 2-rigidity families are $\mathcal{R}_2$ and $\mathcal{H}_2$.

Theorem (Tyomkyn 2026+)

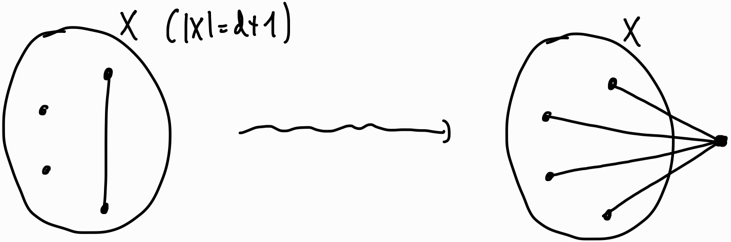
The only 2-rigidity family $\mathcal{M}$ for which $K_{3,3}$ is $\mathcal{M}$-rigid is $\mathcal{M} = \mathcal{R}_2$.

Major conjecture

$$\mathcal{R}_3 = \mathcal{C}_2^1.$$

Extensions

The d-**dimensional** 1-**extension** operation:



Definition

We say that a d-dimensional graph matroid family $\mathcal{M}$ is **1-extendable** if d-dimensional 1-extension preserves $\mathcal{M}$-independence.

For example: $\mathcal{R}_d$, $\mathcal{M}_{k,\ell}$ are 1-extendable, but $\mathcal{E}$, $\mathcal{H}_d$ are not.

Theorem (G.-Villányi 2026+)

For a d-dimensional graph matroid family $\mathcal{M}$,

$K_{d+1,k}$ is $\mathcal{M}$-rigid for some $k \Rightarrow \mathcal{M}$ is 1-extendable.

Theorem (G.-Villányi 2026+)

For a d-dimensional graph matroid family $\mathcal{M}$,

$$K_{d+1,k} \text{ is } \mathcal{M}\text{-rigid for some } k \Rightarrow \mathcal{M} \text{ is } 1\text{-extendable.}$$

Moreover, if $\mathcal{M}$ is a d-rigidity family, then

$$K_{d+1,\binom{d+1}{2}} \text{ is } \mathcal{M}\text{-rigid} \Leftrightarrow \mathcal{M} \text{ is } 1\text{-extendable.}$$

Theorem (G.-Villányi 2026+)

For a d-dimensional graph matroid family $\mathcal{M}$,

$$K_{d+1,k} \text{ is } \mathcal{M}\text{-rigid for some } k \Rightarrow \mathcal{M} \text{ is } 1\text{-extendable.}$$

Moreover, if $\mathcal{M}$ is a d-rigidity family, then

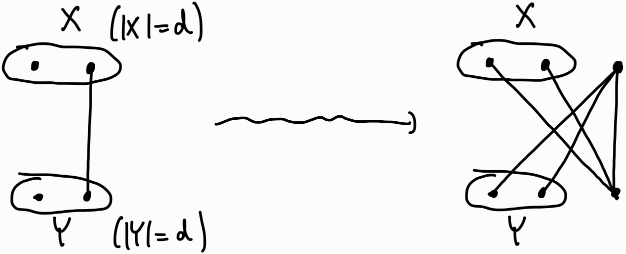
$$K_{d+1,\binom{d+1}{2}} \text{ is } \mathcal{M}\text{-rigid} \Leftrightarrow \mathcal{M} \text{ is } 1\text{-extendable.}$$

Corollary

For a 2-rigidity family $\mathcal{M}$, the following are equivalent:

- $K_{3,3}$ is $\mathcal{M}$-rigid,
- $\mathcal{M}$ is 1-extendable,
- $\mathcal{M} = \mathcal{R}_2$.

The d-**dimensional double** 1-**extension** operation:



When $X \neq Y$, this can be modeled as performing two consecutive 1-extensions.

Definition

We say that a d-dimensional graph matroid family $\mathcal{M}$ is **double 1-extendable** if d-dimensional double 1-extension preserves $\mathcal{M}$-independence.

Theorem (G.-Villányi 2026+)

Every graph matroid family is double 1-extendable.

Theorem (G.-Villányi 2026+)

Every graph matroid family is double 1-extendable.

Theorem (G.-Villányi 2026+)

Let $\mathcal{M}$ be a graph matroid family with dimension $d \geq 2$, and let G be a connected $(d+1)$-regular graph. If $G \neq K_{d+2}, K_{d+1,d+1}$, then G is $\mathcal{M}$-independent.

(The case when $\mathcal{M}$ is a 2-rigidity family was proved by Tyomkyn.)

Connectivity

Theorem (Lovász-Yemini 1982)

Every 6-connected graph is $\mathcal{R}_2$-rigid.

Theorem (Lovász-Yemini 1982)

Every 6-connected graph is $\mathcal{R}_2$-rigid.

Theorem (Nash-Williams 1961, Tutte 1961)

Every $2k$-edge-connected graph contains k edge-disjoint spanning trees.

Theorem (Lovász-Yemini 1982)

Every 6-connected graph is $\mathcal{R}_2$-rigid.

Theorem (Nash-Williams 1961, Tutte 1961)

Every $2k$-edge-connected graph is $\mathcal{M}_{k,k}$-rigid.

Theorem (Lovász-Yemini 1982)

Every 6-connected graph is $\mathcal{R}_2$-rigid.

Theorem (Nash-Williams 1961, Tutte 1961)

Every $2k$-edge-connected graph is $\mathcal{M}_{k,k}$-rigid.

Theorem (Clinch-Jackson-Tanigawa 2022)

Every 12-connected graph is $\mathcal{C}_2^1$-rigid.

Theorem (Lovász-Yemini 1982)

Every 6-connected graph is $\mathcal{R}_2$-rigid.

Theorem (Nash-Williams 1961, Tutte 1961)

Every $2k$-edge-connected graph is $\mathcal{M}_{k,k}$-rigid.

Theorem (Clinch-Jackson-Tanigawa 2022)

Every 12-connected graph is $\mathcal{C}_2^1$-rigid.

Theorem (Villányi 2025)

Every $d(d+1)$-connected graph is $\mathcal{R}_d$-rigid.

Definition

A family $\mathcal{M}$ has the **Lovász-Yemini property** if there is $c \geq 0$ such that every c-connected graph is $\mathcal{M}$-rigid.

Definition

A family $\mathcal{M}$ has the **Lovász-Yemini property** if there is $c \geq 0$ such that every c-connected graph is $\mathcal{M}$-rigid.

Note: the family $\mathcal{E}$ of even cycle matroids does not have the Lovász-Yemini property. ($K_{m,n}$ is never $\mathcal{E}$-rigid.)

Definition

A family $\mathcal{M}$ has the **Lovász-Yemini property** if there is $c \geq 0$ such that every c-connected graph is $\mathcal{M}$-rigid.

Note: the family $\mathcal{E}$ of even cycle matroids does not have the Lovász-Yemini property. ($K_{m,n}$ is never $\mathcal{E}$-rigid.)

Theorem (Villányi 2025, G. 2026)

Every 1-extendable graph matroid family has the Lovász-Yemini property.

The converse is open.

$K_{d+1,k}$ is $\mathcal{M}$-rigid for some k

$\Downarrow$

$\mathcal{M}$ is 1-extendable

$\Downarrow$

$\mathcal{M}$ has the Lovász-Yemini property.

$$K_{d+1,k} \text{ is } \mathcal{M}\text{-rigid for some } k$$
$$\Downarrow$$
$$\mathcal{M} \text{ is 1-extendable}$$
$$\Downarrow$$
$$\mathcal{M} \text{ has the Lovász-Yemini property.}$$

Theorem (G.-Villányi 2026+)

Equivalent for a graph matroid family $\mathcal{M}$:

- $\mathcal{M}$ has the Lovász-Yemini property,
- $K_{m,n}$ is $\mathcal{M}$-rigid for some $m, n \geq d+1$.

A nice corollary

Let $\mathcal{R}_d^k$ be the k-fold union of $\mathcal{R}_d$.

This has the Lovász-Yemini property (it is 1-extendable).

A nice corollary

Let $\mathcal{R}_d^k$ be the k-fold union of $\mathcal{R}_d$.

This has the Lovász-Yemini property (it is 1-extendable).

Lemma

An $\mathcal{R}_d^k$-rigid graph on at least $2kd$ vertices contains k edge-disjoint $\mathcal{R}_d$-rigid spanning subgraphs.

A nice corollary

Let $\mathcal{R}_d^k$ be the k-fold union of $\mathcal{R}_d$.

This has the Lovász-Yemini property (it is 1-extendable).

Lemma

An $\mathcal{R}_d^k$-rigid graph on at least $2kd$ vertices contains k edge-disjoint $\mathcal{R}_d$-rigid spanning subgraphs.

Combining these, we get:

Theorem (G.-Jordán-Király-Villányi 2025)

Every sufficiently highly connected graph contains k edge-disjoint $\mathcal{R}_d$-rigid spanning subgraphs.

Theorem (G.-Jordán-Király-Villányi 2025)

Every $(k \cdot 10d(d+1))$-connected graph contains k edge-disjoint $\mathcal{R}_d$-rigid (and hence d-connected) spanning subgraphs.

Previously it was even open whether we can find, in highly connected graphs, a d-connected spanning subgraph whose complement is connected.

Further directions

- Analogs of Whitney's theorem for graph matroid families.
- Constructing new d-rigidity families.
- Coning and duality.
- Extensions to multigraphs, directed graphs, hypergraphs...

Thank you!

Some reading:

- Kalai, **Symmetric matroids**, *JCTB*, 1990.
- Garamvölgyi, **Rigidity and reconstructions in matroids of highly connected graphs**, *JCTB*, 2026.
- Tyomkyn, **On plane rigidity matroids**, 2026. arXiv:2602.11892
- A paper with Soma Villányi, eventually..